DIGITAL SIMULATION OF THE LINKS OF THERMAL IDENTIFICATION SYSTEMS

A. G. Shashkov, V. P. Kozlov,
and G. P. Pisarik

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Methods of digital simulation of the links with irrational and transcendental transfer functions are studied. The simulation errors are presented for different values of the variable parameters.

The problem of constructing thermal identification measurement-information systems is linked with the need to construct models of links with irrational $1/(b\sqrt{p})$ and transcendental $\exp(-k\sqrt{p})$ transfer functions [1]. These links are described by partial differential equations, so that active, distributed RC structures or their discrete equivalents in a definite frequency range are employed for their analog simulation [2]. Because of the large time constant of the thermal processes, however, in order to develop analog models it is necessary to have RC structures with large time constants, which cannot always be realized in practice with the required accuracy. The striving to increase the accuracy of the simulation led to the development of digital models of thermal links. They can be constructed based on a digital computer or a specialized digital device. For the identify criterion it is convenient to chosse the magnitude of the deviation of the output signals of the digital model and of the thermal link for the same inputs. The digital model operates with a digital signal. In order to represent a continuous thermal process in a digital form, it is necessary to carry out the discretization operation in time and quantization by levels [3]. For a correctly chosen discretization frequency there is no loss of information about the continuous signal and the signal can be reconstructed exactly from the discrete measurements. The discretization and reconstruction operations are mutually inverse, if the continuous signal has a bounded spectrum F_M and the discretization frequency F_e satisfies Kotel'nikov's theorem [4]

$$F_e \geqslant 2F_M. \tag{1}$$

To increase the accuracy of the representation of the continuous signal the discretization frequency is chosen from more rigid conditions [3]

$$F_e \geqslant (5-10)\, F_M. \tag{2}$$

The quantization operation is nonlinear and introduces an unavoidable error into the representation of the signal. However, when a large enough number of quantization levels is chosen the rms value of the quantization noise is small $\bar{\varepsilon}^2 = \delta\Theta^2/12$ and the analog signal can be replaced by the digital signal. The digital model is described by a discrete transfer W(z). In order to form the discrete equivalent of a continuous transfer function W(p) it is necessary to know its Z transform $W(z)=Z\{W(p)\}$ and it must be represented in the form of a recurrence difference equation, which forms the basis for the construction of the algorithm for processing the digital signal [3]. The described method for determining the discrete equivalent is predicated on the assumption that the continuous transfer function is described by a rational expression. Thermal links, however, are characterized by irrational $1/(b\sqrt{p})$ and transcendental $\exp(-k\sqrt{p})$ continuous transfer functions, which cannot be described by rational expressions with the required accuracy. Therefore in order to construct a digital model of a thermal link we shall employ a discrete convolution. In this case the response of the digital model to an arbitrary input in the time domain is defined as

$$\hat{\Theta}_{\text{out}}(nT_e) = \sum_{l=0}^{n} \hat{\Theta}_{\text{in}}(lT_e)\, \hat{h}[(n-l)\,T_e] = \sum_{l=0}^{n} \hat{\Theta}_{\text{in}}[(n-l)\,T_e]\, \hat{h}(lT_e).$$

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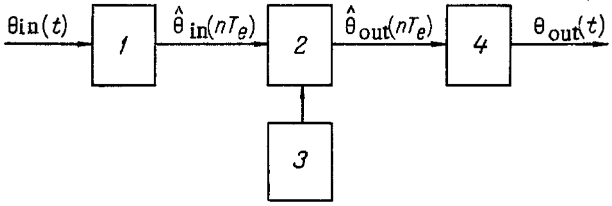

Fig. 1. Structural layout of the digital model based on a special convolution processor: 1) analog to digital converter; 2) special convolution processor; 3) weighting function working memory; 4) digital-to-analog converter.

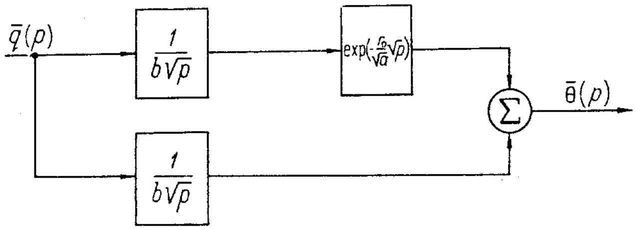

Fig. 2. Structural diagram of the formation of the temperature field at the center of a round heat source.

In this case the convolution in the form of a sum is understood to be not the approximation to the convolution integral, but rather as a means for determining the n-th value of the output sequence of the digital model. Employing for the impulsive characteristic $\hat{h}(lT_e)$ the weighting function of the thermal link, represented in digital form, it is possible to realize the equivalent digital model.

A digital computer or a special convolution processor is employed in order to implement the digital model. A structural diagram of the digital model based on a special convolution processor is shown in Fig. 1. The weighting function of the thermal link is stored in digital form in the working memory (WM) of the weighting function. The special processor provides the convolution of an arbitrary input signal and a weighting function, and as a result a digital signal whose value is identical to the value of the output signal of a continuous thermal link at the time $t = nT_e$ (for identical inputs) is formed at the output. The output sequence can be formed with the use of the algorithm for sectioned convolution.

We shall examine as an example the structural characteristics of the digital model of a link with the transfer function $\exp(-k\sqrt{p})$, employed in the system for identifying the coefficients of thermal diffusivity a and thermal activity b without destroying the integrity of the material under study. The system is constructed with the help of the structural scheme for heat transport in a semibounded body, whose surface is exposed to a bounded circular heat source with radius r_0 and specific power q(t) (Fig. 2) [5]. The mathematical model of the body under study is represented by links with the transfer function $1/(b\sqrt{p})$ and $\exp(-k\sqrt{p})$. The numerical values of the coefficient of thermal diffusivity a for most of the nonmetallic materials studied i.e., in the following intervals: $a = (0.5\text{–}10)\cdot 10^{-7}$ m^2/sec. Taking into account the radius of the circular heat source the range of possible values $k = (6.7\text{–}67)$ sec$^{1/2}$. The weighting function h(t) of the link $\exp(-k\sqrt{p})$ is given by

$$h(t) = \frac{k}{2\sqrt{\pi t^3}} \exp(-k^2/4t).$$

For $t < 0$ $h(t) \equiv 0$, so that the Fourier transform of the weighting function corresponds to its Laplace transform at $p = j\omega$

TABLE 1. Values of the Output Signals with an Input 1/p

Time t, sec	Output signal		Error δ, %
	digital model	thermal link	
	$k=6{,}7$		
3	$0{,}5795\cdot10^{-3}$	$0{,}8082\cdot10^{-3}$	28
4	$0{,}5950\cdot10^{-2}$	$0{,}6233\cdot10^{-2}$	4,5
5	$0{,}1764\cdot10^{-1}$	$0{,}1784\cdot10^{-1}$	1,1
6	$0{,}3399\cdot10^{-1}$	$0{,}3411\cdot10^{-1}$	0,35
7	$0{,}5304\cdot10^{-1}$	$0{,}5310\cdot10^{-1}$	0,1
8	$0{,}7333\cdot10^{-1}$	$0{,}7335\cdot10^{-1}$	0,03
9	$0{,}9393\cdot10^{-1}$	$0{,}9393\cdot10^{-1}$	0,00
10	0,1143	0,1143	0,00
	$k=67$		
10	$0{,}3489\cdot10^{-57}$	$0{,}3721\cdot10^{-55}$	—
20	$0{,}1110\cdot10^{-26}$	$0{,}1662\cdot10^{-26}$	—
30	$0{,}1301\cdot10^{-17}$	$0{,}1418\cdot10^{-17}$	8,20
40	$0{,}3222\cdot10^{-13}$	$0{,}3321\cdot10^{-13}$	2,99
50	$0{,}1294\cdot10^{-10}$	$0{,}1313\cdot10^{-10}$	1,43
60	$0{,}6894\cdot10^{-9}$	$0{,}6950\cdot10^{-9}$	0,81
70	$0{,}1172\cdot10^{-7}$	$0{,}1178\cdot10^{-7}$	0,5
80	$0{,}9796\cdot10^{-7}$	$0{,}9830\cdot10^{-7}$	0,34
90	$0{,}5113\cdot10^{-6}$	$0{,}5125\cdot10^{-6}$	0,34
100	$0{,}1921\cdot10^{-5}$	$0{,}1924\cdot10^{-5}$	0,17

TABLE 2. Values of the Output Signals with the Input $1/(p\sqrt{p})$

Time t, sec	Output signal		Error δ, %
	digital model	thermal link	
	$k=6{,}7$		
3	$0{,}4624\cdot10^{-3}$	$0{,}4198\cdot10^{-3}$	10
4	$0{,}5086\cdot10^{-2}$	$0{,}4625\cdot10^{-2}$	9,98
5	$0{,}1778\cdot10^{-1}$	$0{,}1618\cdot10^{-1}$	5,23
6	$0{,}4001\cdot10^{-1}$	$0{,}3884\cdot10^{-1}$	3,00
7	$0{,}7137\cdot10^{-1}$	$0{,}7006\cdot10^{-1}$	1,88
8	0,1107	0,1094	1,26
9	0,1568	0,1554	0,89
10	0,2084	0,2071	0,66
11	0,2645	0,2632	0,5
12	0,3243	0,3230	0,35
13	0,3869	0,3857	0,31
14	0,4518	0,4507	0,25
15	0,5186	0,5175	0,21
16	0,5869	0,5858	0,17
	$k=67$		
80	$0{,}2185\cdot10^{-6}$	$0{,}2033\cdot10^{-6}$	7,51
90	$0{,}1273\cdot10^{-5}$	$0{,}1213\cdot10^{-5}$	4,98
100	$0{,}5278\cdot10^{-5}$	$0{,}5103\cdot10^{-5}$	3,43
110	$0{,}1706\cdot10^{-4}$	$0{,}1666\cdot10^{-4}$	2,43
120	$0{,}4575\cdot10^{-4}$	$0{,}4496\cdot10^{-4}$	1,76
130	$0{,}1062\cdot10^{-3}$	$0{,}1043\cdot10^{-3}$	1,30
140	$0{,}2198\cdot10^{-3}$	$0{,}2177\cdot10^{-3}$	0,98
150	$0{,}4154\cdot10^{-3}$	$0{,}4123\cdot10^{-3}$	0,75
160	$0{,}7283\cdot10^{-3}$	$0{,}7242\cdot10^{-3}$	0,57
170	$0{,}1200\cdot10^{-2}$	$0{,}1195\cdot10^{-2}$	0,45
180	$0{,}1879\cdot10^{-2}$	$0{,}1873\cdot10^{-2}$	0,35
190	$0{,}2816\cdot10^{-2}$	$0{,}2807\cdot10^{-2}$	0,28
200	$0{,}4063\cdot10^{-2}$	$0{,}4054\cdot10^{-2}$	0,22

$$H(j\omega)=[\exp(-k\sqrt{p})]_{p=j\omega}=\exp(-k\sqrt{j\omega}),$$

$$|H(j\omega)|=\exp\left(-\frac{k}{\sqrt{2}}\sqrt{\omega}\right).$$

The weighting function h(t) has a continuous unbounded spectrum. In order to obtain a correct discretization of h(t) we shall limit the width of the spectrum by the value ω_M and we

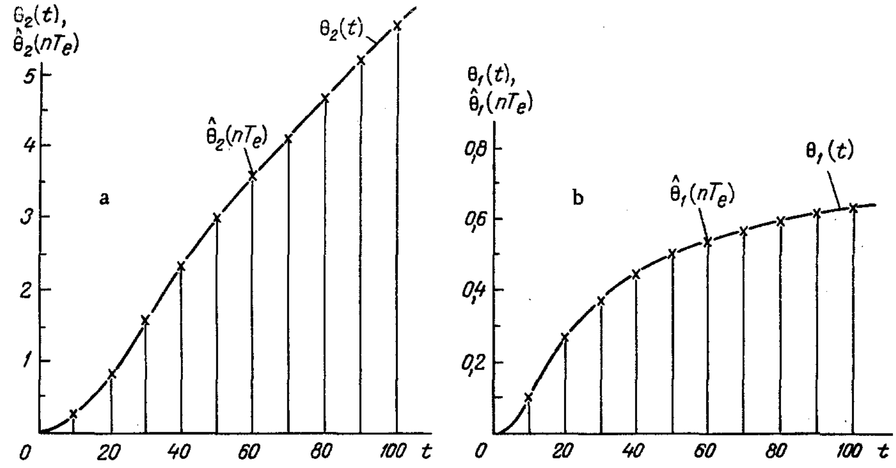

Fig. 3. Graphs of the output signals of the thermal link and digital model for inputs $1/(p\sqrt{p})$ (a) and $1/p$ (b).

shall determine the error introduced by dropping the high-frequency components as follows:

$$\varepsilon_b = \frac{H(j\omega) - H(j\omega_M)}{|H(j\omega)|} \cdot 100\% = \exp\left(-\sqrt{2k^2\omega_M}\right)\left[1 + \sqrt{2k^2\omega_M}\right] \cdot 100\%.$$

For $k \equiv 6.7$ and $\omega_M = 1$ Hz the error does not exceed 0.1%. The effect of the filtering frequencies on the weighting function can be evaluated with the help of the Parseval relation [6].

$$\int_{-\infty}^{\infty} |h(t)|^2 dt = \int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega.$$

We shall determine the discretization frequency F_e from the condition (1): $F_e \geqslant 2F_M = 0.3$ Hz. To raise the accuracy of the simulation we shall increase F_e, according to (2), up to 1 Hz. The weighting function h(t) is determined for all values of t, but is studied in a bounded interval [0, T], which distorts its spectrum and introduces a truncation error. The error introduced by truncating h(t) is given by

$$\varepsilon = \left[\operatorname{erf}\left(\frac{k}{2\sqrt{T}}\right)\right] \cdot 100\%.$$

The size of the interval T can be chosen so that outside an interval of duration T the value of h(t) would not exceed β % of its maximum value $h_{max}(t)$:

$$h(t > T) \leqslant \beta\% h_{max}(t).$$

The weighting function h(t) has a maximum at $t = k^2/6$. The size of the interval T for $k = 6.7$, $\beta < 1\%$ equals 250 sec.

To evaluate the accuracy of the simulation and to determine more accurately the observation interval T a complex of numerical experiments was performed for different values of k. The experiments consisted of calculating the output signal of the digital model using the algorithm for sectioned convolution and comparing it with the output signal of the link $\exp(-k\sqrt{p})$ at the time $t = nT_e$. The functions $1/p$ and $1/(p\sqrt{p})$, characteristic for systems used to identify the thermophysical properties of materials, were employed at the inputs. The values of the output signals of the thermal link for the indicated inputs were determined, respectively, by the formulas

$$\hat{\Theta}_1(nT_e) = \operatorname{erfc}\left(\frac{k}{2\sqrt{nT_e}}\right),$$

$$\hat{\Theta}_2(nT_e) = 2\sqrt{\frac{nT_e}{\pi}}\exp(-k^2/4nT_e) - k\operatorname{erfc}\left(\frac{k}{2\sqrt{nT_e}}\right).$$

As the results of the numerical experiments showed, for the input 1/p the maximum deviation of the output signals of the digital model and the thermal link is observed initially (Table 1). For t > 7 sec (k = 6.7) the error does not exceed 0.1%. Increasing k decreases the output signals initially and therefore extends the initial section up to 100 sec. For t > 100 sec the error does not exceed 0.17%. For the input $1(p\sqrt{p})$ the error of the digital model is initially somewhat higher (Table 2). But for t > 15 sec (k = 6.7) it does not exceed 0.2%, For k = 67, t = 200 sec the error does not exceed 0.22%. Graphs of the output signals of the thermal link and digital model for inputs $1/(p\sqrt{p})$ and 1/p are represented in Fig. 3. For k = 6.7 the output signals of the digital model and thermal link are identical in the observation interval [10, 100 sec].

NOTATION

p, Laplace transform parameter; $\delta\Theta$, quantization step for the excess temperature Θ; $\Theta(t)$, excess temperature, $\hat{\Theta}_{out}$, is the digital signal at the input of the model, n = 0, 1, 2 ..., $\operatorname{erfc} x = 1 - \operatorname{erf} x$, $\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2)\, dx$ is Gauss'error function.

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RECONSTRUCTION OF CAUSAL CHARACTERISTICS OF THE THERMAL CONDUCTIVITY PROCESS FROM THE SOLUTION OF THE COMBINED INVERSE PROBLEM

O. M. Alifanov, S. L. Balakovskii,
and M. V. Klibanov

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An algorithm is suggested for solving the combined inverse problem of heat exchange on the basis of using uniqueness theorems.

Methods of inverse problems of heat exchange (IPHE), substantially enhancing the effectiveness of studies in this region, have become widely used in analyzing heat-exchange processes. Among the various IPHE formulations, one can distinguish the combined methods [1], when one seeks simultaneously causal characteristics of various types. Thus, in simulating thermal processes is heat-protection materials, during plasma deposition, heating, and a number of other cases the necessity arises of determining the thermal conductivity coefficient in the high temperature region. At the same time the low measurement accuracy does not make it possible to obtain reliable information concerning external thermal loads and internal heat sources, related, for example, to chemical reaction flow in the bulk of the material investigated. This difficulty is overcome as a result of solving the combined IPHE, consisting of determining the coefficients of the thermal conductivity equation and the heat flux density at the boundary from temperature measurements at internal points of the body. We note that in several cases the temperatures at internal points are the only reliable source of information on the thermal state of the object.

V. V. Kuibyshev Polytechnic Institute.

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